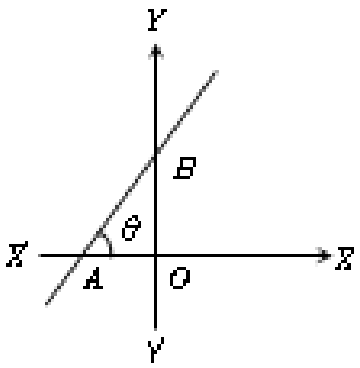
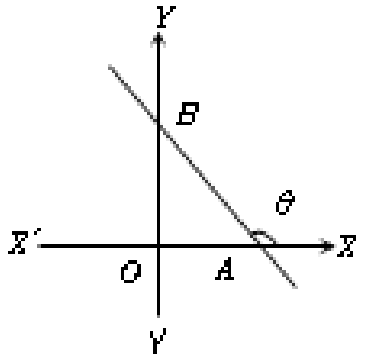
Slope (Gradient) of a line

The trigonometrical tangent of the angle that a line makes with the positive direction of the x-axis in anticlockwise sense is called the slope or gradient of the line. The slope of a line is generally denoted by m. Thus, $m = \tan\theta$.





(1) Slope of line parallel to x – axis is $m = \tan 0^o = 0$.

(2) Slope of line parallel to y – axis is $m = \tan 90^o = \infty$.

(3) Slope of the line equally inclined with the axes is 1 or – 1.

(4) Slope of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$ taken in the same order.

(5) Slope of the line$ax + by + c = 0, b \neq 0$ is $-\frac{a}{b}$.

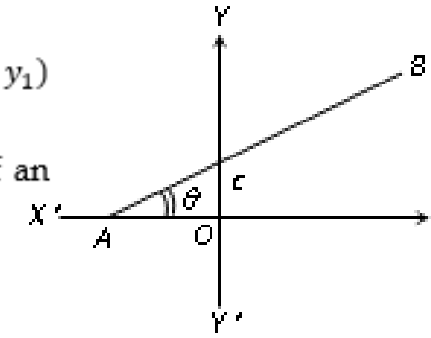
(6) Slope of two parallel lines are equal.

(7) If m_1 and m_2 be the slopes of two perpendicular lines, then $m_1 . m_2 = -1$.

(8) m can be defined as $\tan\theta$ for $0 \leq \theta \leq \pi$ and $\theta \neq \frac{\pi}{2}$.

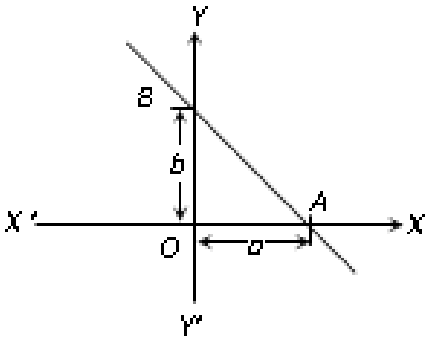
Equations of straight line in different forms

1. **Slope form:** Equation of a line through the origin and having slope m is $y = mx$.
2. **One point form or Point slope form:** Equation of a line through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.
3. **Slope intercept form:** Equation of a line (non-vertical) with slope m and cutting off an intercept c on the y-axis is $y = mx + c$.



The equation of a line with slope m and the x-intercept d is $y = m(x - d)$

4. **Intercept form:** If a straight-line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.
   Then, equation of a straight line cutting off intercepts a and b on x–axis and y–axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.



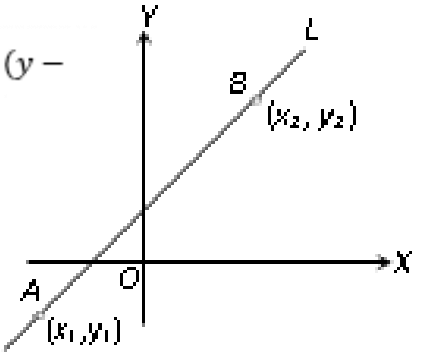
- ★ If given line is parallel to X axis, then X-intercept is undefined.
- ★ If given line is parallel to Y axis, then Y-intercept is undefined.

5. **Two point form:** Equation of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is, $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

In the determinant form it is gives as

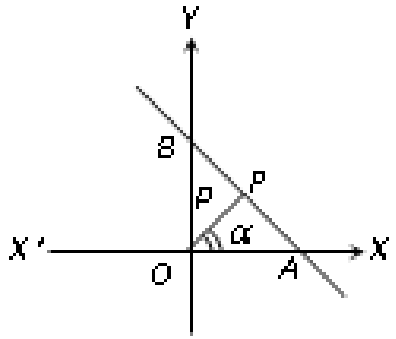
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

is the equation of line.



Equations of straight line in different forms

6. Normal or perpendicular form: The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with x-axis is $x\cos\alpha + y\sin\alpha = p$.



7. Symmetrical or parametric or distance form of the line: Equation of a line passing through (x_1, y_1) and making an angle θ with the positive direction of x-axis is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = \pm r$, where r is the distance between the point $P(x, y)$ and $A(x_1, y_1)$.

The co-ordinates of any point on this line may be taken as $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$, known as parametric co-ordinates. 'r' is called the parameter.